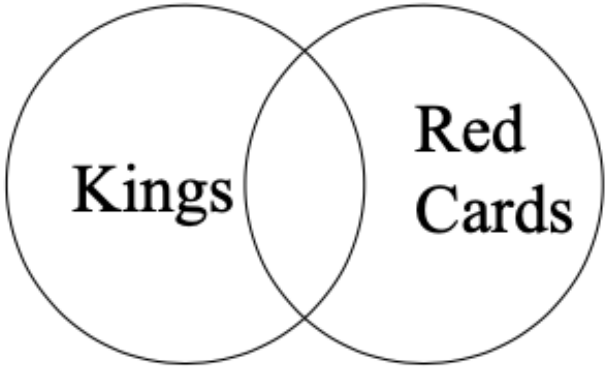
Conditional Probability

10.2 Independent and Dependent Events

Conditional Probability



One card is drawn from a deck.

$$P(king \mid red\ card) = \frac{red\ kings}{all\ red\ cards} = \frac{2}{26} = \frac{1}{13}$$

10.2 Independent and Dependent Events

Independent vs. Dependent events

One card is drawn from a deck.
Is the event “Jack” and “Heart” independent?

$$P(jack) = \frac{4}{52} = \frac{1}{13}$$

$$P(jack \mid heart) = \frac{1}{13}$$

Independent!!!

10.2 Independent and Dependent Events

Independent vs. Dependent events

One card is drawn from a deck.
Is the event “club card” and “black card” independent?

$$P(club) = \frac{13}{52} = \frac{1}{4}$$

$$P(club \mid black) = \frac{13}{26} = \frac{1}{2}$$

Dependent!!!

Binomial Distributions

10.6 Binomial Distributions

Binomial Trials

A coin is tossed 3 times. What is the probability that it will be 3 heads?

$$P(3\ heads) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

Asel normally wins 1 out of every 3 chess games (P = 1/3). Suppose she plays 4 games. What is the probability that she will win 3 games and lose one?

Ways she can have 3W and 1L:

____ ____ ____ ____

WWWL WWLW
WLWW LWWW

$${}_4C_3 = 4$$

$$P(3W) = {}_4C_3 \cdot W^3L^1 = 4 \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1$$

$$= \frac{8}{81}$$

10.6 Binomial Distributions

Binomial Trials

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$(a+b)^0$ 1
$(a+b)^1$ 1 1
$(a+b)^2$ 1 2 1
$(a+b)^3$ 1 3 3 1
$(a+b)^4$ 1 4 6 4 1

$$(W+L)^4 = W^4L^0 + \boxed{4W^3L^1} + 6W^2L^2 + 4W^1L^3 + W^0L^4$$

10.6 Binomial Distributions

Binomial Trials

Asel normally wins 1 out of every 3 chess games (P = 1/3). Suppose she plays 4 games. What is the probability that she will win 3 games and lose one?

$$P(3W) = {}_4C_3 \cdot W^3L^1 = 4 \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 = \frac{8}{81}$$

Practice -

1. Matthew and Anirud play 5 games. The probability that Matthew wins is 3/7. What is the probability that Matthew will win 3 games?

$$P(3W) = {}_5C_3 \cdot W^3L^2$$

$$P(3W) = 10 \cdot \left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^2 = \frac{4{,}320}{16{,}807}$$

10.6 Binomial Distributions

Binomial Trials

Harmony and Melody play 7 games. The probability that Harmony wins is 1/5. What is the probability that Harmony will win <u>at least</u> 3 games?

$$(x+y)^n = \sum_{k=0}^{n} {}_nC_k \cdot x^{n-k}y^k$$

$(a+b)^0$ 1
$(a+b)^1$ 1 1
$(a+b)^2$ 1 2 1
$(a+b)^3$ 1 3 3 1
$(a+b)^4$ 1 4 6 4 1
$(a+b)^5$ 1 5 10 10 5 1
$(a+b)^6$ 1 6 15 20 15 6 1
$(a+b)^7$ 1 7 21 35 35 21 7 1

$$(W+L)^7 = W^7L^0 + 7W^6L^1 + 21W^5L^2 + 35W^4L^3 + 35W^3L^4 + 21W^2L^5 + 7W^1L^6 + W^0L^7$$

$$P(at\ least\ 3W) = \frac{11{,}565}{78{,}125} = \frac{2{,}313}{15{,}625}$$

10.6 Binomial Distributions

Binomial Trials

Harmony and Melody play 7 games. The probability that Harmony wins is 1/5. What is the probability that Harmony will win <u>at least</u> 3 games?

$$(x+y)^n = \sum_{k=0}^{n} {}_nC_k \cdot x^{n-k}y^k$$

$$(W+L)^7 = \boxed{W^7L^0 + 7W^6L^1 + 21W^5L^2 + 35W^4L^3 + 35W^3L^4} + 21W^2L^5 + 7W^1L^6 + W^0L^7$$

Practice -

1. Aditya and Saumit play chess. Aditya's probability of winning is 3/5. If they play 5 matches, then what is the probability of Aditya winning 3 or more matches?

$$(W+L)^5 = \boxed{W^5L^0 + 5W^4L^1 + 10W^3L^2} + 10W^2L^3 + 5W^1L^4 + W^0L^5$$

$$P(3 \text{ or more } W) = \left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^0 + 5\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^1 + 10\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2 = \frac{243+810+1080}{3125} = \frac{2{,}133}{3{,}125}$$

10.6 Binomial Distributions

When do I use Binomial Distributions vs P and C?

a) When there are exactly two outcomes, like win/lose, heads/tails.

b) When there is a fixed number of trials.

c) When each trial is independent of the other.

d) When the probability for success is always the same.

1) Two cards are chosen. What is the probability that they are both spades?

$$P(2\ spades) = \frac{{}_{13}C_2}{{}_{52}C_2} = \frac{78}{1326} = \frac{1}{17}$$

2) Sid wins at poker 1/5 of the time. What is the probability he will win 3 out of 5 games?

$$P(W3\ of\ 5) = {}_5C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^2 = 0.0512$$